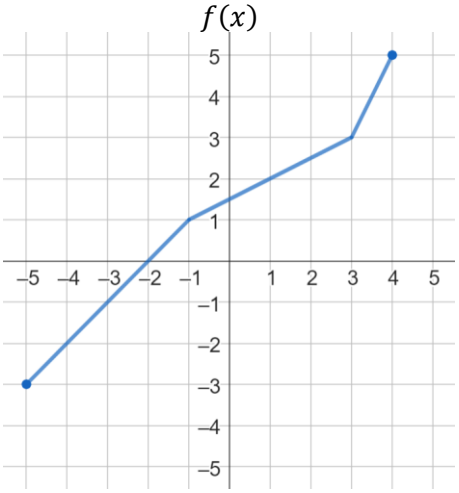
Inverse Functions (2.8) Homework

Use the graph of $f(x)$, equation of $g(x)$, and table of $h(x)$ values to evaluate the expression in questions #1-9 and to select an answer choice to #10.



$g(x) = 4x - 1$

x	$h(x)$
-5	-10
-4	-7
-3	-6
-2	-3
-1	4
0	1
1	0
2	8
3	6
4	2
5	3

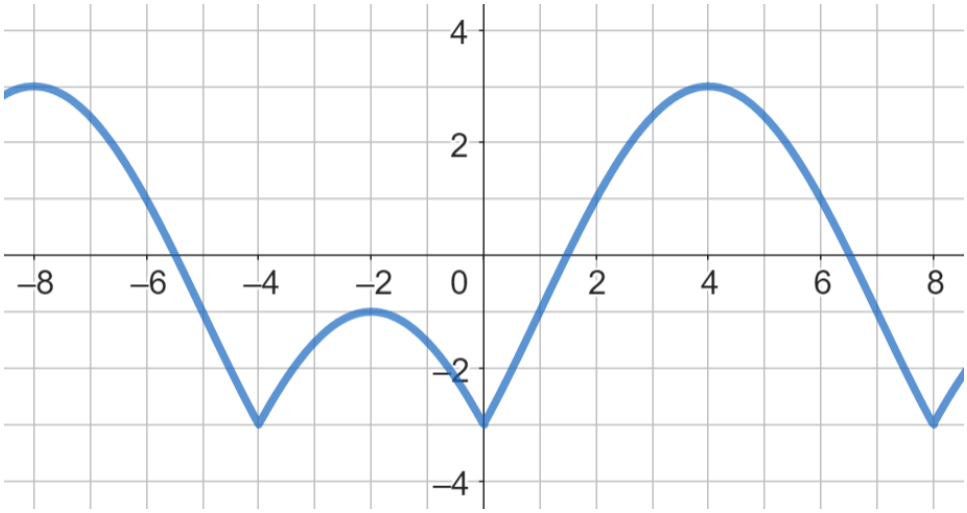
1. $f^{-1}(2)$
2. $g^{-1}(5)$
3. $h^{-1}(6)$
4. $f\left(h^{-1}(2)\right)$
5. $(g \circ f^{-1})(-2)$
6. $(g^{-1} \circ h)(2)$
7. $(h \circ h^{-1})(4)$
8. $(g^{-1} \circ f^{-1})(-1)$
9. $(g \circ f)^{-1}(-13)$
10. The table does not give a value for $h(6)$. For which one of the following values of $h(6)$ is $h(x)$ a one-to-one function?

    (A) $h(6) = -2$

    (B) $h(6) = 0$

    (C) $h(6) = 2$

    (D) $h(6) = 4$
11. Let $f(x) = \frac{2x-9}{3}$ for all real values of x. Find $f^{-1}(x)$.
12. Let $g(x) = \frac{3}{x-4} + 5$ for all $x \neq 4$. Find $g^{-1}(x)$.
13. Let $h(x) = \sqrt[3]{2x-5} + 1$ and $k(x) = \frac{(x-1)^3+5}{2}$. Verify that h and k are inverses of one another.
14. Let $p(x) = \frac{3}{x-2}$ and $q(x) = \frac{3}{x} + 2$. Verify algebraically that p and q are inverses of one another.



15. Consider the graph of $y = f(x)$ shown above. Select all of the following domain restrictions that yield an invertible function.

(A) $y = f(x)$ for $-8 < x < -4$

(B) $y = f(x)$ for $-4 < x < 0$

(C) $y = f(x)$ for $0 < x < 4$

(D) $y = f(x)$ for $-4 < x < -2$

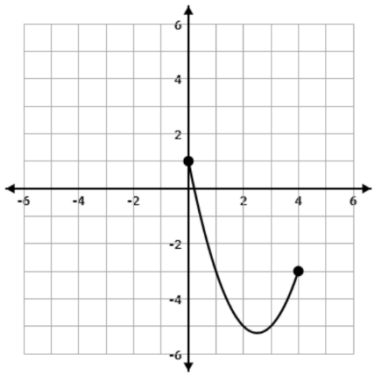
(E) $y = f(x)$ for $-1 < x < 1$

(F) $y = f(x)$ for $1 < x < 3$

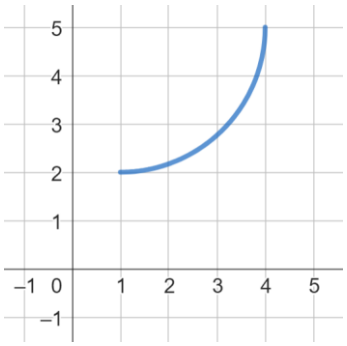
16. Scientist have studied leatherback sea turtles, the royalty of the sea turtle world, and discovered they typically do most their growth in the first fifteen years after hatching. Full size for a leatherback sea turtle is on average 2000 lbs in weight and 6 ft in length. Let S be a function that models the size of a leatherback during its first fifteen years after hatching. S inputs the age of a leatherback sea turtle and outputs the turtle's weight.
    a. True or false?: S^{-1} inputs the weight of a leatherback and outputs a number that may be used to predict the age of the turtle.
    b. True or false?: S^{-1} is likely a decreasing function.
    c. Predict the domain of S^{-1}
    d. Predict the range of S^{-1}

17. Consider the function $f(x) = \frac{1}{3}x^3 + 2x^2 + \frac{7}{4}x - 1$
    a. Use Desmos or a graphing calculator to solve $f(x) = 2$. Round your answers to three decimal places.
    b. Over what interval or intervals does $f(x)$ increase?
    c. f is not invertible. Let $g(x) = f(x)$ with the restricted domain $x \geq -\frac{1}{2}$. g is invertible. Find $g^{-1}(2)$.
    d. Find $g^{-1}(3)$.
    e. Find the x-intercept of $g^{-1}(x)$.
    f. Find the y-intercept of $g^{-1}(x)$.

18. The figure to the right shows the graph of the function g on its domain $0 \leq x \leq 4$.
    a. Determine if g has an inverse function.
    b. Give a reason for your answer based on the definition of a function and the graph of $y = g(x)$.

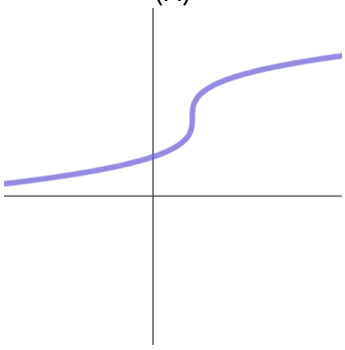


19. Consider $f(x)$ graphed to the right, defined over $1 \leq x \leq 4$.
    a. What is the maximum of $f^{-1}(x)$?
    b. Is the rate of change of f^{-1} positive or negative?
    c. Is the rate of change of f^{-1} increasing or decreasing?

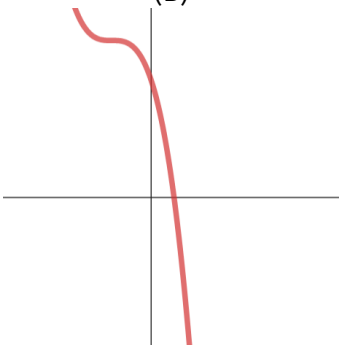


20. Below are six functions, each labelled with a letter. The inverse of each function is one of the other given functions. Divide these six functions into three pairs of two based on which functions are inverses of each other.

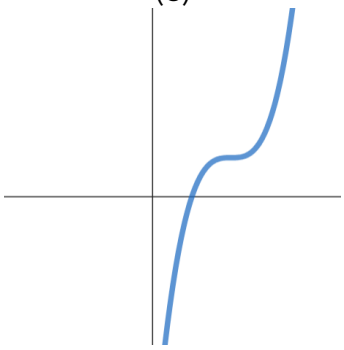
(A)



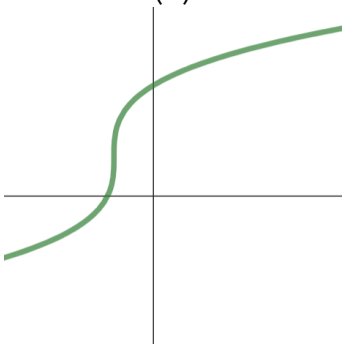
(B)



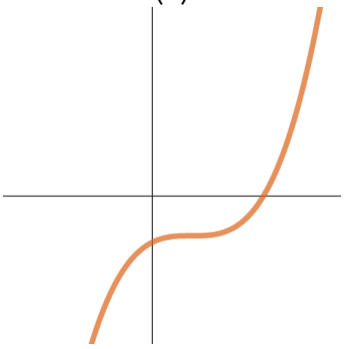
(C)



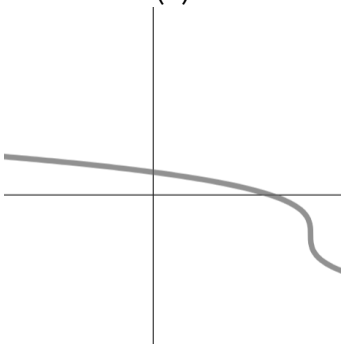
(D)



(E)



(F)



Inverse Functions HW Answers

1. 1
2. $\frac{3}{2}$
3. 3
4. 5
5. -17
6. $\frac{9}{4}$
7. 4
8. $-\frac{1}{2}$
9. -5
10. A
11. $f^{-1}(x) = \frac{3x+9}{2}$
12. $g^{-1}(x) = \frac{3}{x-5} + 4$ for all $x \neq 5$
13. $h(k(x)) = h\left(\frac{(x-1)^3+5}{2}\right) = \sqrt[3]{2\left(\frac{(x-1)^3+5}{2}\right) - 5} + 1 = \sqrt[3]{((x-1)^3+5) - 5} + 1 = \sqrt[3]{(x-1)^3} + 1 = x - 1 + 1 = x$

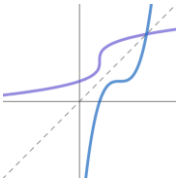
    $k(h(x)) = k(\sqrt[3]{2x-5} + 1) = \frac{\left(\left(\sqrt[3]{2x-5}+1\right)-1\right)^3+5}{2} = \frac{\left(\sqrt[3]{2x-5}\right)^3+5}{2} = \frac{2x-5+5}{2} = \frac{2x}{2} = x$

    Since $(h \circ k)(x)$ and $(k \circ h)(x)$ both equal x, h and k are inverses.
14. $p(q(x)) = p\left(\frac{3}{x} + 2\right) = \frac{3}{\left(\frac{3}{x}+2\right)-2} = \frac{3}{3/x} = x$

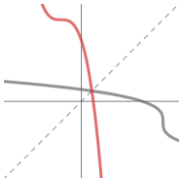
    $q(p(x)) = q\left(\frac{3}{x-2}\right) = \frac{3}{\frac{3}{x-2}} + 2 = x - 2 + 2 = x$

    Since $(p \circ q)(x)$ and $(q \circ p)(x)$ both equal x, p and q are inverses.
15. A, C, D, and F
16.
    a. True. If S does age→weight, then S^{-1} does weight→age.
    b. False. If S^{-1} were a decreasing function, then plugging in larger outputs would result in smaller outputs. It is highly unlikely that a good model would associate a larger weight with a younger turtle when pertaining to the part of a turtle's lifespan in which it grows.
    c. $[0\text{ lb}, 2000\text{ lb}]$. The domain of S^{-1} = The range of S = The set of turtle weights over the fifteen years of growth. If you said something along the lines of $[1\text{ oz}, 2000\text{ lb}]$, you are even more correct. No turtle will have a weight of zero.
    d. $[0\text{ yrs}, 15\text{ yrs}]$. The range of S^{-1} = The domain of S. The question said the model S was for "its first fifteen years after hatching."
17.
    a. $x = -4.284, x = -2.542, x = 0.826$
    b. f increases on the interval $(-\infty, -3.5]$ and on the interval $[-0.5, \infty)$
    c. $g^{-1}(2) = 0.826$
    d. $g^{-1}(3) = 0.988$
    e. $(-1,0)$
    f. $(0, 0.388)$
18. a. No b. Since $g(2) = g(3)$, g does not have an inverse function.
19. a. 4 b. Positive c. Decreasing
20.

A and C are inverses of each other



B and F are inverses of each other



D and E are inverses of each other